DELAY LINE GEOMETRY (ver. 5: 4 Nov. 2001)

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Abstract

IOTA delay line geometry is derived for the 2- and 3-telescope configurations. Nominal values of some delay line constants are listed. Section 5 is new as of November 2001.

1 TELESCOPE VECTOR

Suppose we have telescopes at locations $\vec{T}_i, i = 1, 2, 3$. To be precise, $\vec{T}$ points to the intersection of the rotational axes of an ideal telescope. For a star that is being tracked, this is the point along the ray path at which the direction cosines of the wavefront switch from time-varying to constant.

Each telescope vector $\vec{T}$ can be expressed as components along the x (east), y (north), and z (up) axes, or as a distance d and direction (θ, ϕ) from the corner position of the array:

$$\vec{T} = (x, y, z) \tag{1}$$

$$= (east, north, up) \tag{2}$$

$$= d \times (\sin\theta\cos\phi, \cos\theta\cos\phi, \sin\phi) \tag{3}$$

where $d = |\vec{T}|$, θ is the azimuth, and ϕ is the elevation. A plan view of the IOTA telescope stations and an example of connecting baselines is shown in Figure 1.

The station distances d are multiples of the lengths of steel beams along the arms. The "5 m" stations, nominally at (5, 10, 15, 20, 25, 30) m, are at multiples of 197 inch = 500.38 cm. The "7 m" stations, nominally at (7, 14, 21, 28, 35) m, are at multiples of 277 inch = 703.58 cm (ref. Traub, "Station Locations", 29 Nov 96).

Table of explicit values to be inserted here.

The orientations of the NE and SE arms are nominally

$$(\theta, \phi)_{NE} = (+41.0292^{o}, -0.0082^{\circ}) \tag{4}$$

$$(\theta, \phi)_{SE} = (+131.0292^{\circ}, +0.0082^{\circ}) \tag{5}$$

i.e., the array is rotated slightly CCW from a NE-SE orientation, and the northerly stations are slightly lower than the southerly stations. These values were determined by sighting with a theodolite from the 15 m stations to Polaris, and assuming a 90° angle between arms, equal distances to the corner, and a uniform tilt overall (ref. Traub, “IOTA Orientation”, 6 May 95).

More precise station locations can be determined from the corresponding baselines by least-squares fitting of WLP observations across the sky.

Telescopes are numbered clockwise from north, as listed in this Table.

number	letter	name
1	A	North
2	B	East
3	C	West

The terrestrial location of IOTA was originally determined from a USGS map. These coordinates were used from 1993-1998 in the DelayLine.Config file, listed in the following Table. Note that $1'' = 31$ m $= 0.0003^\circ$ latitude, so a $1''$ error circle includes nearly the entire IOTA instrument.

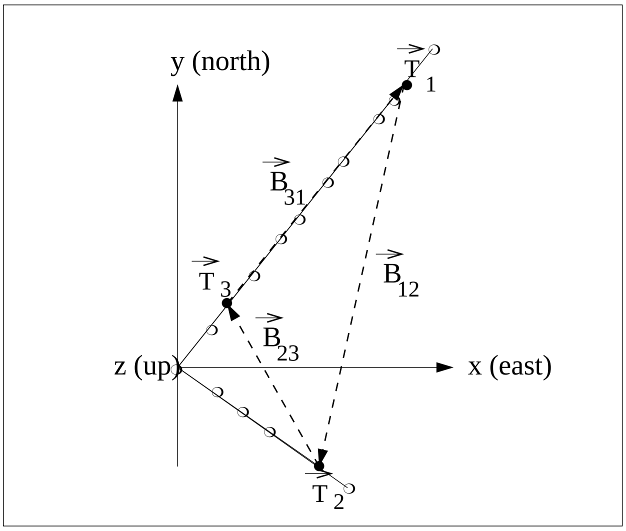

Figure 1: Schematic plan view of IOTA, showing available telescope stations (open circles), 3 sample occupied stations (filled circles) labeled $\vec{T}_i$, connecting baseline vectors $\vec{B}_{ij}$, and the $(x, y, z) = (East, North, Up)$ coordinate system with origin at the corner station.

coordinate (map)	value (common units)	value (decimal)
west longitude	$110^\circ 53' 04.00''$	110.884444°
geodetic latitude	$31^\circ 41' 20.00''$	31.688889°

We adapt the following definitions from the Astronomical Almanac. Terrestrial longitude is the equatorial angle from Greenwich to the local meridian. Geodetic latitude is the meridional angle from the equator the local geodetic vertical. The geodetic vertical points to the astronomical zenith, which is the extension to infinity of a plumb line, so it is perpendicular to the free surface of water. The quantities we want for IOTA are terrestrial longitude (so we can calculate the hour angle of a star from our geographic location), geodetic latitude (so we can calculate the declination distance of a star with respect to the local vertical), and geodetic altitude (so we can estimate the mean air pressure). Note that if the star position program also calculates the elevation and azimuth of the star, the latitude value used should be geodetic latitude.

Since 1998 an on-site GPS receiver has been used to determine IOTA's coordinates. We allow the receiver to continuously integrate its signal for about 1 day, to reduce the dithering ("selective availability") error. Potential errors include: the receiver is located in the lab, not at the corner of the array; the absolute accuracy of the receiver is unknown; the error as a function of integration time is unknown; and the repeatability of the measurement is unknown. Our current best location data from GPS is the following Table.

coordinate (GPS)	value (common units)	value (decimal)
west longitude	$110^{\circ}53'05.73''$	110.884925°
geodetic latitude	$31^{\circ}41'29.58''$	31.691550°
geodetic altitude	8412 ft	2564 m

The longitude difference between (map - GPS) is about $2''$, equivalent to $2/15 \simeq 0.1$ sec of time. The latitude difference is about $10''$. Both differences are significantly larger than the site size. It would be intellectually satisfying (but not crucial) to reduce these uncertainties in the future.

2 BASELINE VECTOR

The baseline vector $\vec{B}_{ij}$ points from $\vec{T}_i$ to $\vec{T}_j$, so that

$$\vec{B}_{ij} = \vec{T}_j - \vec{T}_i \tag{6}$$

as shown in Figure 1. In general there are two types of baseline to consider. One type is needed to calculate the wavefront delay (cf. w below), and this is set by the vector from the intersecting axes of one telescope to the intersecting axes of another. The other type is needed to calculate the mutual coherence of the wavefront (for which we need u, v, below), and this depends on the shape of the aperture, and whether the wavefront is sheared; for a pair of telescopes with identical apertures, these types of baseline are identical, but otherwise they are not necessarily the same.

The real-time delay-tracking program uses the (xyz) components of $\vec{B}_{ij}$ directly, so no further work is needed to provide nominal first-guess values.

However in the tracking program we need to project $\vec{B}_{ij}$ first onto an Earth-centered system, then onto the sky. Specifically, in the (east, north, up) system that we use at IOTA, we have

$$BE_{ij} = x_i - x_j \quad (7)$$

$$BN_{ij} = y_i - y_j \quad (8)$$

$$BU_{ij} = z_i - z_j \quad (9)$$

Suppose we define an Earth-oriented coordinate system (X, Y, Z), where the (X, Y) plane is parallel to the Earth's equator (i.e., the $\delta = 0$ plane), the X axis is in the meridian plane through the Earth's rotation axis and the corner of the array, the Y axis is toward the east, and the Z axis is parallel to the Earth's rotation axis (ref. TMS, p.89). In other words, we simply need to rotate $\vec{B}_{ij}$ from its local geodetic coordinates to an Earth centered system.

In the $(X, Y, Z))$ system, the baseline vector has components

$$X = BU \cos\lambda - BN \sin\lambda \quad (10)$$

$$Y = BE \quad (11)$$

$$Z = BU \sin\lambda + BN \cos\lambda \quad (12)$$

where λ is the geodetic latitude of the corner of the array, and we have dropped the subscripts for convenience.

3 STAR VECTOR

The star position is $(RA, DEC) = (\alpha, \delta)$, referred to the Earth's axis of rotation and zero-point of rotation at the current epoch, including precession, nutation, and polar motion, and also annual aberration and possibly diurnal aberration, but *not* including atmospheric refraction.

We exclude refraction from the delay calculation because, to first order, the Earth's atmosphere is static and plane parallel. Static implies that the seeing and piston fluctuations are negligible. Plane parallel implies identical wavefront delays at each telescope, so the common-mode delay can be ignored, since we are only interested in relative delays.

Note that the *pointing* direction of the telescopes *should* include a nominal refractive correction. If ζ_{vac} is the zenith angle of the star above the atmosphere, ζ_{tel} the zenith angle at the telescope, and n the index of refraction of air at the telescope, then $n \sin\zeta_{tel} = \sin\zeta_{vac}$ gives the correct zenith angle for the telescope pointing. This can be simplified (Allen, 3rd ed, p.124) for the altitude at IOTA and for a wavelength of about 1μm to give

$$\zeta_{tel} = \zeta_{vac} - 44 \tan\zeta_{vac} \quad \text{(arcsec)}. \quad (13)$$

This correction should be included in the actual and displayed elevation angles of the telescope, and also in any off-line fitting procedure designed to solve for the telescope internal parameters (the Pointing Model).

Let us denote by $\hat{s}$ the unit vector from the telescope to the phase center of the star (TMS, p.80). The sky projection of $\vec{B}$ at the location of the target star gives baseline components (u, v, w) where

$$u = \text{proj. of } \vec{\mathrm{B}} \text{ perp. to } \hat{\mathrm{s}}, \text{ parallel to } +\alpha \text{ (RA) axis} \quad (14)$$

$$v = \text{proj. of } \vec{\mathrm{B}} \text{ perp. to } \hat{\mathrm{s}}, \text{ parallel to} + \delta \text{ (DEC) axis} \quad (15)$$

$$w = \text{proj. of } \vec{\mathrm{B}} \text{ parallel to} + \hat{\mathrm{s}}. \quad (16)$$

Thus $+u$ points east on the sky, $+v$ points north on the sky, and $+w$ points into the plane of the sky (Figure 2) .

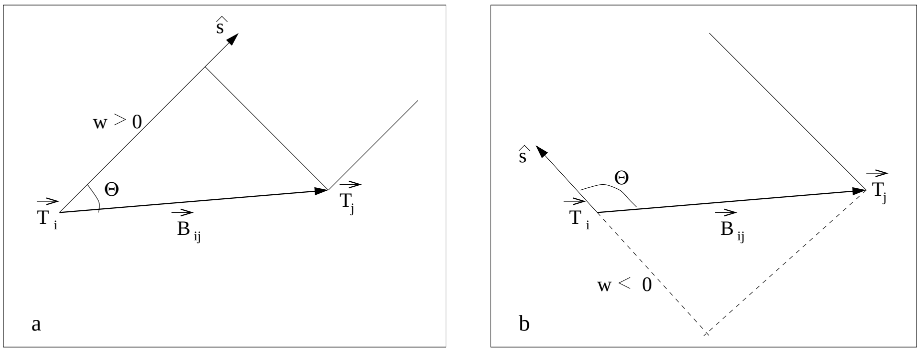

Figure 2: The angle Θ from the baseline vector $\vec{B}$ to the star vector $\hat{s}$ can be (a) acute, so that the external path difference w is positive, or (b) obtuse, so that w is negative.

The (u, v, w) components of $\vec{B}$ can be calculated (ref TMS, p.86) from the Earth-oriented baseline vector (X, Y, Z), in cm units, and the star position (α, δ).

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (17)$$

where H is the hour angle.

As the star moves and H varies, the magnitude of the (u, v) vector gives the projected baseline $\vec{B}^{\perp}_{\lambda}$ in units of wavelengths (or cycles/radian), as

$$\vec{B}^{\perp}_{\lambda} = \sqrt{u^2 + v^2}/\lambda \quad (18)$$

or in units of cycles/arcsec, as

$$\vec{B}^{\perp}_{\mathrm{cy/as}} = \sqrt{u^2 + v^2}/(206, 265\lambda). \quad (19)$$

The locus of points is an ellipse in the (u, v) plane (TMS, p.89).

Let us denote by OPD(ext) the optical path difference of rays external to the baseline $\vec{B}_{ij}$ of a pair of telescopes. By construction this is

$$OPD(ext) = w. \tag{20}$$

If the external wavefront reaches $\vec{T}_i$ last (see Figure 2), then $w \geq 0$ and the external delay is positive. If the external wavefront reaches $\vec{T}_j$ last, then $w \leq 0$ and the external delay is negative.

In other words, consider the two vectors $\vec{B}_{ij}$ and $\vec{s}$ originating at $\vec{T}_i$, and the angle Θ between them, where $\cos\Theta = \hat{B}_{ij} \cdot \vec{s}$. If $\Theta = (0, \pi/2)$ then $w \geq 0$; if $\Theta = (\pi/2, \pi)$ then $w \leq 0$. See Figure 2.

It is sometimes useful to know the rate of change of w, which is seen to be

$$dw/dt = -u \cos\delta dH/dt \tag{21}$$

4 INTERNAL DELAY

Let us denote by $OPD(int)$ the optical path difference of the internal rays, given by the path length L_j from $\vec{T}_j$ to the combination point $\vec{O}$ minus the path L_i from $\vec{T}_i$ to $\vec{O}$,

$$OPD(int) = L_j - L_i \tag{22}$$

where each path length is the sum of segments between reflections

$$L_j = \sum_{\vec{O}}^{\vec{T}_j} \Delta L_j \tag{23}$$

$$L_i = \sum_{\vec{O}}^{\vec{T}_i} \Delta L_i. \tag{24}$$

Maximum fringes are detected when $OPD(int)$ is adjusted to the white light point (WLP), and this occurs when the total OPD is zero

$$OPD(ext) + OPD(int) = 0. \tag{25}$$

We assume that the siderostats axes at $\vec{T}$ are mechanically perfect, in the sense that the mirror should tilt about a point in its surface. Departures from this behavior can be modeled separately, and may be found as a part of the empirical pointing vector intrinsic to each individual telescope.

We assume that the internal paths L_i and L_j are composed of vacuum plus equal amounts of glass or air in each arm, the mirror reflection angles have the same order and magnitude in each arm, the polarization effects are identical, and the beamsplitter has no effect on the phase of the transmitted and reflected beams. Schematic paths are shown in Figure 3.

There are 2 delay line systems at IOTA, labeled (LD1, SD1) and (LD2, SD2), which are the original and more recent installations, respectively. There is a corresponding pair of *fixed* mirrors called the *stub delay* path. Switching mirrors direct the beams as needed.

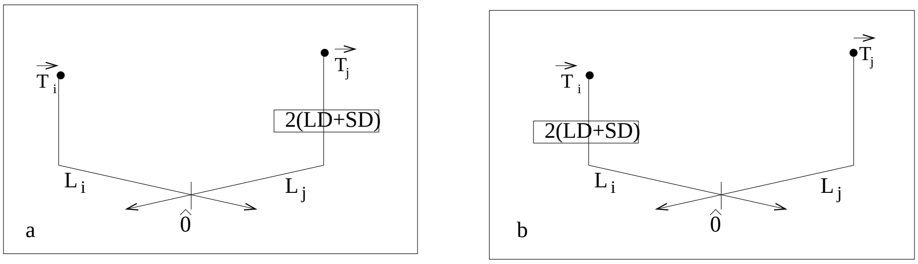

Figure 3: The total internal path from each telescope $\vec{T}$ to the combiner $\hat{O}$ is given by L, including the delay paths $2(LD+SD)$, for (a) $\vec{T_j}$ internally delayed, or (b) $\vec{T_i}$ internally delayed.

In principle, there are 6 possible combinations of 3 telescopes and 3 types of delay line. However in practice, and as shown elsewhere (TBD), we expect that only 2 combinations are needed, the first for the star generally north

lab output	internal path	telescope
fixed	stub delay	$\vec{T_2}$
delay 1	(LD1, SD1)	$\vec{T_1}$
delay 2	(LD2, SD2)	$\vec{T_3}$

and the second for the star generally south

lab output	internal path	telescope
fixed	stub delay	$\vec{T_1}$
delay 1	(LD1, SD1)	$\vec{T_2}$
delay 2	(LD2, SD2)	$\vec{T_3}$

The net result is that only 2 switching mirrors are required, since the 3rd telescope beam always goes to its own dedicated delay line. (In this sense, the present memo is more general than necessary.)

The internal coordinates (details TBD) of the delay lines are given in this Table (details TBD). Note that SD1 and SD2 are in mechanical series on a singleair table, such that at their home positions, SD1 is about 41 cm northeast of SD2, where 41 cm is the length of SD2 plus about 2 cm to allow for the homing action of SD1. In other words, when both are at home, they are at the southeast extreme of the air table, separated by about 2 cm. In practice SD1 must be moved away from home before SD2, in order that they not collide. However collisions should be automatically prevented by the VxW software.

delay line	home (cm)	extreme (cm)
LD1	0.0	+2800.0
LD2	0.0	+2800.0
SD1	0.0	-200.0
SD2	0.0	-200.0

Delay line positions are measured with HP interferometers to a precision of 1/60 wave $\simeq 0.01\mu$m. For visual convenience, these positions should be displayed to 4 decimal places, giving a display precision of 1μm = 0.0001 cm.

For a given baseline $\vec{B}_{ij}$ and computed external delay w_{ij}, if the j-beam needs an internal delay (Figure 3), then we have

$$\begin{aligned} L_i &= L_i^0(int.del.j) &(26)\\ L_j &= L_j^0(int.del.j) + 2(LD + SD) &(27)\\ OPD(int) &= L_j - L_i &(28)\\ &= C_{ij}(int.del.j) + 2(LD + SD) &(29) \end{aligned}$$

where $C_{ij}(int.del.j) = L_j^0(int.del.j) - L_i^0(int.del.j)$ is a constant. To acquire fringes, we require

$$OPD(ext) + OPD(int) = 0. \tag{30}$$

For the j-beam internal delay, this yields

$$w_{ij} + C_{ij}(int.del.j) + 2(LD + SD) = 0 \tag{31}$$

which tells us where to place LD and how to move SD.

Likewise, if the i-beam needs an internal delay (Figure 3), then we have

$$\begin{aligned} L_i &= L_i^0(int.del.i) + 2(LD + SD) &(32)\\ L_j &= L_j^0(int.del.i) &(33)\\ OPD(int) &= L_j - L_i &(34)\\ &= C_{ij}(int.del.i) - 2(LD + SD) &(35) \end{aligned}$$

where $C_{ij}(int.del.i) = L_j^0(int.del.i) - L_i^0(int.del.i)$ is a constant. For the i-beam internal delay, this yields

$$w_{ij} + C_{ij}(int.del.i) - 2(LD + SD) = 0. \tag{36}$$

To summarize, we need

$$\boxed{LD + SD = -\tfrac{1}{2}[w_{ij} + C_{ij}(int.del.j)] \quad \text{... if int.del. j}} \tag{37}$$

$$\boxed{LD + SD = +\tfrac{1}{2}[w_{ij} + C_{ij}(int.del.i)] \quad \text{... if int.del. i}} \tag{38}$$

Nominal values of C_{ij} are determined by tape measure for a particular station in each arm, and extended to other stations using values of d, where in general $L_i^0 = const + d_i$, so we get

$$C_{ij} = C_{ij}^0 + (d_j - d_i). \tag{39}$$

5 Note on Post-July-2001 Delays

This section is copied from my 8 Nov. 2001 memo to John Monnier at IOTA, who found internal fringes with SD1 and the G3 using these values. (Addendum to "Delay Line Geometry (25 Feb 2000)" Memo)

In general $OPD(int)+OPD(ext) = 0$ is needed for fringes. Now $OPD(ext) = w$ is calculated by the machine, along with u, v. We supply $OPD(int)$ from measurements.

To simplify the system of measurement, we now split the internal path into 2 major segments (I, from telescope to the lab; E, from the lab to the beam-combiner of each specific experiment, as follows.

In general

$$OPD_{ij}(int) = I_j - I_i + E_j - E_i \tag{40}$$

where I_i is the Interferometer path from telescope i to the lab, and E_i is the continuation of path I_i from the lab to the beam combiner for that particular Experiment. Here "the lab" means a single plane perpendicular to all beams as they emerge from the windowed pipes in the laboratory. Also, tel.1 is A, 2 is B, 3 is C, of course.

My measurements give I_1, I_2, I_3. Each experiment must supply its own values of E_1, E_2, E_3.

On the basis of tape-measure values on 1 July 2001, and for the specific case of just two telescopes, T1 = "35m" and T2 = "15m", I find

$$OPD_{12}(int, delaytel1) = C_{12}(deltel1) - 2(LD1 + SD1) \tag{41}$$

$$OPD_{12}(int, delaytel2) = C_{12}(deltel2) + 2(LD1 + SD1) \tag{42}$$

where

$$C_{12}(deltel1) = -2104.54cm + E_2 - E_1 \tag{43}$$

$$C_{12}(deltel2) = -1828.95cm + E_2 - E_1 \tag{44}$$

If the Fluor G3 is used to control SD1, then it uses the "magic constant" MC where MC = C - 39.65 cm, so

$$MC_{12}(deltel1) = -2144.19cm + E_2 - E_1(starnorth) \tag{45}$$

$$MC_{12}(deltel2) = -1868.60cm + E_2 - E_1(starsouth). \tag{46}$$

Note the nominal ranges are

$$LD(1or2) = 0 \text{ to} + 2800cm \tag{47}$$

$$SD(1or2) = 0 \text{ to} - 200cm. \tag{48}$$

6 APPENDIX

(*The material in this appendix was cut from the text to make version 5, and will be discarded later. It is retained here for possible reference value only.)*

For the classical infrared table and the visible table we find nominal values

$$C_{12}(int.del.1)^0 = -104.6\text{cm} \quad (49)$$
$$C_{12}(int.del.2)^0 = +203.2 \quad (50)$$

The 3rd-telescope values needed are

$$C_{23}(int.del.1)^0 = tbd \quad (51)$$
$$C_{31}(int.del.2)^0 = tbd \quad (52)$$

For historical reasons, the Fluor program uses the Fluor "magic constant" MC which differs from C_{ij} by the "Fluor offset"

$$FO = 39.65 \text{ cm} \quad (53)$$
$$C = MC + FO. \quad (54)$$

The table gives some current measured C and MC values. A single entry in column 6 is a C value, and an entry with "+FO" is an MC value.

Combiner	Target	d_1 (m)	d_2 (m)	int.del.	C_{12} or $MC + FO$ (cm)	Source
classical IR	sky	15	15	1	−104.6	WT 10/99 & notes
classical IR	sky	35	15	1	−2121.4	WT 10/99 & notes
classical IR	sky	15	15	2	+203.2	WT 10/99 & notes
classical IR	sky	35	15	2	−1814.1	WT 10/99 & notes
FLUOR	sky	15	15	1	+312.7+FO	RMG/JDM 2/00, drift
FLUOR	sky	25	15	1	−688.6+FO	RMG/JDM 2/00
FLUOR	autocol	15	15	1	+309.6+FO	RMG/JDM 2/00, drift
FLUOR	autocol	35	15	1		RMG/JDM 2/00
FLUOR	sky	21	5	1?	−1300.4+FO	GP/VF $\leq$ 6/99
FLUOR	sky	21	15	1?	−299.6+FO	GP/VF $\leq$ 6/99
FLUOR	sky	25	15	1?	−690.8+FO	GP/VF $\leq$ 6/99

In two cases, once on the sky and once in autocollimation, a distinctive drift of MC values was noted, over several hours, from 313.1 to 312.4 (-0.7 cm), and again 309.9 to 309.4 (-0.5 cm). This drift makes recent values uncertain by several mm.

In autocoll, with hollow corner cubes resting on the ends of the "stovepipes", the C or MC value is about +2.7 to +2.9 cm more positive than the sky value, due to slight differences in the distances to the stovepipe windows versus the siderostats.

For the 2/2000 FLUOR session, and using the 15, 15 value of MC, and the equation to calculate other nominal C or MC values, we compute the following Table. Column and row heads are nominal station positions in meters. Table entries are MC values in cm.

	5 m	7 m	10 m	14 m	15 m
0 m	813.1	1016.3	1313.5	1719.9	1813.8
5 m	312.7	515.9	813.1	1219.5	1313.5
7 m	109.5	312.7	609.9	1016.3	1110.3
10 m	-187.7	15.5	312.7	719.1	813.1
14 m	-594.1	-390.9	-93.7	312.7	406.7
15 m	-688.1	-484.9	-187.7	218.7	312.7
20 m	-1188.4	-985.2	-688.1	-281.7	-187.7
21 m	-1297.7	-1094.5	-797.3	-390.9	-296.9
25 m	-1688.8	-1485.6	-1188.4	-782.0	-688.1
28 m	-2001.2	-1798.0	-1500.9	-1094.5	-1000.5
30 m	-2189.2	-1986.0	-1688.8	-1282.4	-1188.4
35 m	-2704.8	-2501.6	-2204.4	-1798.0	-1704.1

Note that the $15, 15$ entry was set equal to the $2/00$ data, and that the $25, 15$ measured and predicted values (-688.6 and -688.1 cm) are reasonably similar, showing that the Table does have approximately the expected predictive power.

In a later draft of this memo, I will add more tables of C and MC values, and explicit B vector components. I will also comment on calculting a best-fit baseline vector to observed interferogram data.