1 The Baseline Vector

The baseline vector $\vec{\mathsf{b}}$ is defined as the vector from telescope B to telescope A measured in centimeters. Establish a coordinate system with its origin at the corner of the array such that the z axis runs parallel to the long arm through telescope A, the y axis runs parallel to the short arm through telescope B, and the x axis is perpendicular to both, pointing roughly up. In this coordinate system, the z component of the baseline vector is the distance in centimeters from the corner of the array to telescope A, the y component is the distance from the corner to telescope B, and the x component is zero.

Station locations are multiples of 197 inches (500.38 cm) for 5, 10, 15, 20, 25, and 30 meter locations, or 277 inches (703.58 cm) for 7, 14, 21, 28, 35 meter locations. With this information an approximate value for the baseline vector in the (x, y, z) coordinate system is easily computed. The task is now to convert the baseline to the "topocentric" coordinate system with axes corresponding to up, east, and north.

The long arm of the array points to an azimuth of $\theta = 41°.02919$, and the short arm is assumed to perpendicular to it pointing roughly southeast. The slope from the short arm 15 meter station to the long arm 15 meter station was measured to be $\phi' = -0°.01155$, sloping downward to the north.

First, convert the measured downward slope from the short arm 15 meter station to the long arm 15 meter station into a rotation around the y axis defined above. If we assume that the measured downward slope is the maximum downward slope in any direction from the short arm 15 meter station, then the cosine formula from spherical trigonometry gives the desired relation

$$\cos\phi = \sin^2\phi' + \cos^2\phi' \cos\theta \tag{1}$$

which gives $\phi = -0°.00871$.

Now we can convert the baseline from the (x, y, z) coordinates to topocentric coordinates by first rotating around y by ϕ and then around z by θ

$$\begin{bmatrix} b_U \\ b_E \\ b_N \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & -\sin\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \cos\theta & \sin\theta\sin\phi \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \tag{2}$$

or

$$\begin{bmatrix} b_U \\ b_E \\ b_N \end{bmatrix} = \begin{bmatrix} 0.75438 & 0.00015 & -0.00011 \\ 0.65644 & 0.75438 & -0.00010 \\ 0.00015 & 0.00000 & 1.00000 \end{bmatrix} \cdot \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \tag{3}$$

1.1 Optical Path Differences

Optical Path Difference OPD is defined to be the distance travelled by the light from the star through telescope B to the beamsplitter minus the distance travelled from the star through telescope A to the beamsplitter. The External Optical Path Difference OPD_{EXT} is the distance travelled from the star to telescope B minus the distance travelled from the star to telescope A. Since the baseline vector $\vec{\mathsf{b}}$ is defined as going from telescope B to telescope A, the inner product of the baseline vector and a unit vector pointing to the star $\hat{\mathsf{s}}$ is the External Optical Path Difference

$$\mathrm{OPD_{EXT}} = \vec{\mathsf{b}} \cdot \hat{\mathsf{s}} \tag{4}$$

The Internal Optical Path Difference $\mathrm{OPD_{INT}}$ is the distance travelled from telescope B to the beamsplitter minus the distance travelled from telescope A to the beamsplitter. The delay line positions are defined such that increasing the delay line positions always increases the distance from the corresponding telescope to the beamsplitter. Therefore, if the delay lines are in the beam from telescope A, then increasing the delay line positions decreases $\mathrm{OPD_{INT}}$; whereas if the delay lines are in the beam from telescope B, then increasing the delay line positions increases $\mathrm{OPD_{INT}}$. With both delay lines at their home positions, a star at zenith does not balance the interferometer due to a constant term in $\mathrm{OPD_{INT}}$ designated $\mathrm{OPD_{INT0}}$:

$$\mathrm{OPD_{INT}} = \mathrm{OPD_{INT0}} \mp 2 \times (\mathrm{SD} + \mathrm{LD}) \tag{5}$$

where SD and LD are the short and long delay line positions, respectively, and the upper sign applies when the beam from telescope A runs through the delay lines and the lower sign applies when the beam from telescope B runs through the delay lines. The factor of two arises since the beam traverses each delay line twice.

The value of $\mathrm{OPD_{INT0}}$ depends on the locations of the two telescopes and on which telescope beam is run over the delay lines. Numerical values for $\mathrm{OPD_{INT0}}$ are estimated as

$$\begin{aligned} \mathrm{OPD_{INT0}} &= \quad -104.6\ \mathrm{cm} \quad + b_y - b_z \ \text{(A delayed)} \\ \mathrm{OPD_{INT0}} &= \quad 206.4\ \mathrm{cm} \quad + b_y - b_z \ \text{(B delayed)} \end{aligned}$$

From the definitions of OPD, $\mathrm{OPD_{EXT}}$, and $\mathrm{OPD_{INT}}$, it is clear that

$$\mathrm{OPD} = \mathrm{OPD_{INT}} + \mathrm{OPD_{EXT}} \tag{6}$$

The condition for balancing the interferometer is that

$$\mathrm{OPD} = 0 \tag{7}$$

or

$$-\vec{\mathsf{b}} \cdot \hat{\mathsf{s}} = \mathrm{OPD_{INT0}} \mp 2 \times (\mathrm{SD} + \mathrm{LD}) \tag{8}$$

thus

$$\mathrm{SD} + \mathrm{LD} = \pm\frac{1}{2}(\vec{\mathsf{b}} \cdot \hat{\mathsf{s}} + \mathrm{OPD_{INT0}}) \tag{9}$$

where once again the upper sign applies when the beam from telescope A runs through the delay lines, and the lower sign applies when the beam from telescope B runs through the delay lines.

1.2 Fitting the Baseline

The formulae given above for the baseline vector are very approximate, and in practice it is necessary to fit a baseline vector to a set of observations. The quantities which can be observed in order to fit the baseline are the positions of the short and long delay lines which balance the interferometer for a given star. If we designate the sum of the long and short delay line positions d, then the last formula in the previous section gives

$$d = \pm\frac{1}{2}\begin{bmatrix} s_U & s_E & s_N & 1 \end{bmatrix}\begin{bmatrix} b_U \\ b_E \\ b_N \\ \mathrm{OPD}_{\mathrm{INT0}} \end{bmatrix} \tag{10}$$

where s_U, s_E, and s_N are the components of the unit topocentric vector $\hat{\mathsf{s}}$ pointing to the star, and b_U, b_E, and b_N are the components of the baseline vector $\vec{\mathsf{b}}$. If n observations of d are made, then the above equation can be augmented to read

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{bmatrix} = \pm\frac{1}{2}\begin{bmatrix} s_{1U} & s_{1E} & s_{1N} & 1 \\ s_{2U} & s_{2E} & s_{2N} & 1 \\ s_{3U} & s_{3E} & s_{3N} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ s_{nU} & s_{nE} & s_{nN} & 1 \end{bmatrix}\begin{bmatrix} b_U \\ b_E \\ b_N \\ \mathrm{OPD}_{\mathrm{INT0}} \end{bmatrix} \tag{11}$$

In general the above equation is only approximately true due to errors in the measurement of the delays d_i. Nonetheless, good estimates for the baseline parameters b_U, b_E, b_N, and $\mathrm{OPD}_{\mathrm{INT0}}$ can be obtained by minimizing the sum of the squared errors, where the error is defined as the difference between the measured delay d_i on the left hand side of the equation above, and the predicted value $\pm\frac{1}{2}(\hat{\mathsf{s}}_i \cdot \vec{\mathsf{b}} + \mathrm{OPD}_{\mathrm{INT0}})$ on the right hand side. For notational convenience, let

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{bmatrix}, \mathbf{S} = \pm\frac{1}{2}\begin{bmatrix} s_{1U} & s_{1E} & s_{1N} & 1 \\ s_{2U} & s_{2E} & s_{2N} & 1 \\ s_{3U} & s_{3E} & s_{3N} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ s_{nU} & s_{nE} & s_{nN} & 1 \end{bmatrix}, \text{and } \mathbf{b} = \begin{bmatrix} b_U \\ b_E \\ b_N \\ \mathrm{OPD}_{\mathrm{INT0}} \end{bmatrix} \tag{12}$$

and thus the quantity to be minimized is

$$|\mathbf{d} - \mathbf{S}\cdot\mathbf{b}|^2 \tag{13}$$

Our task is thus to choose a vector $\mathbf{b}$ such that the vector $\mathbf{S}\cdot\mathbf{b}$, which lies in the range of $\mathbf{S}$, is as close as possible to the vector $\mathbf{d}$, which in general does not lie in the range of $\mathbf{S}$. A necessary and sufficient condition for this to be true is that the error vector $\mathbf{d} - \mathbf{S}\cdot\mathbf{b}$ be perpendicular to the range of $\mathbf{S}$, i.e.

$$\mathbf{S}^{\mathrm{T}}\cdot(\mathbf{d} - \mathbf{S}\cdot\mathbf{b}) = 0 \tag{14}$$

or

$$\mathbf{S}^{\mathrm{T}}\mathbf{S} \cdot \mathbf{b} = \mathbf{S}^{\mathrm{T}} \cdot \mathbf{d}. \tag{15}$$

This equation can be solved formally for the baseline parameters **b**

$$\mathbf{b} = (\mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{T}} \cdot \mathbf{d}. \tag{16}$$

Note that if at least four observations are made, then $\mathbf{S}^{\mathrm{T}}\mathbf{S}$ is a 4×4 symmetric, positive definite matrix whose inverse can be found efficiently by Cholesky decomposition.